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Study on decays of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ in terms of the CQM model

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Abstract. Based on the assumption that $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ are the $(0^+, 1^+)$ chiral partners of D_s and D^*_s, we evaluate the strong pionic and radiative decays of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ in the constituent quark meson (CQM) model. Our numerical results of the relative ratios of the decay widths are reasonably consistent with the data.

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1 Introduction

The new discoveries of exotic particles $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$, which possess spin–parity structures of 0^+, 1^+ respectively [1–3], attract great interest of both theorists and experimentalists of high energy physics. Some authors [4] suppose that $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ are $(0^+, 1^+)$ chiral partners of D_s and D^*_s i.e. p-wave excited states of D_s and D^*_s [5]. Narison used the QCD spectral sum rules to calculate the masses of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ by assuming that they are quark–antiquark states and obtained results which are consistent with the data within a wide error range [6]. Beveren and Rupp also studied the mass spectra [7] and claimed that their results support the $c\bar{s}$ structures for $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$. Meanwhile, some other authors suggest that $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ can possibly be four-quark states [8–11]. Very recently Close and Swanson highlighted several key issues concerning the determination of the properties of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ [12]. Thus, one needs to try various ways to understand the structures of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$. In general, one can take a reasonable theoretical approach to evaluate related physical quantities and then compare the results with the data to extract useful information. One can determine the structures of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ by studying the production rates of the exotic particles, and our recent work [13] is just about the production of $D^*_{sJ}(2317)$ in the decays of $\psi(4415)$.

Recently, several groups have calculated the strong and radiative decay rates of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ in different theoretical approaches: the light cone QCD sum rules, constituent quark model, vector meson dominant (VMD) ansatz, etc. [14–20]. For a clear comparison, the results by different groups are listed in Tables 2 and 3. The authors of [9, 21] also calculated the rates based on the assumption that $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ are in non-$c\bar{s}$ structures. Their predictions on the $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ decay rates are obviously larger than that obtained by assuming the two-quark structure by orders. Thus studies on the strong and radiative decays with other plausible models would be helpful. It cannot only deepen our understanding about the characters of these particles but also test the reliability of models which are applied to calculate the decays. Because $D_{sJ}(2632)$ was only observed by the SELEX collaboration [22], but not by Babar [23], Belle [24] and FOCUS [25], its existence is still in dispute, so here we do not refer to decays of $D_{sJ}(2632)$.

In this work, we study the strong and radiative decays of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ in the framework of constituent quark meson (CQM) model. CQM model was proposed by Polosa et al. [26] and has been well developed later based on the works of Ebert et al. [27] (see [26] for a review). The model is based on an effective Lagrangian which incorporates the flavor–spin symmetry for heavy quarks with the chiral symmetry for light quarks. Employing the CQM model to study the phenomenology of heavy meson physics, reasonable results have been achieved [28, 29]. Therefore, we believe that the model is applicable to our processes and expect to get relatively reliable conclusions.

The constraint from the phase space of the final states forbids the processes $D^*_{sJ}(2317) \to D_s\eta(\eta')$ and $D_{sJ}(2460) \to D^*_s\eta(\eta')$, so that the only allowed strong decay modes are $D^*_{sJ}(2317) \to D_s\pi^0$ and $D_{sJ}(2460) \to D^*_s\pi^0$. In principle, $D_{sJ}(2460) \to D^*_{sJ}(2317) + \pi^0$, which is a $1^+ \to 0^+ + 0^-$ process, is allowed by the phase space. However, it is a p-wave reaction, and the total rate is proportional to $|\mathbf{p}|^2$, where $\mathbf{p}$ is the three-momentum of the emitted pion in the center-of-mass frame of $D_{sJ}(2460)$. In this case $|\mathbf{p}|$ is very small (about $\sim 28\,\mathrm{MeV}$), so that this

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process can only contribute to the total width a negligible fraction, in practice.

The aforementioned strong decay modes obviously violate isospin conservation. Therefore the decay widths of $D_{sJ}^*(2317) \to D_s\pi^0$ and $D_{sJ}(2460) \to D_s^*\pi^0$ must be highly suppressed. Moreover, direct emission of a pion is OZI suppressed [30].

Cho et al. suggested a mixing mechanism of η–π^0 where the isospin violation originates from the mass splitting of u and d quarks [31]. In that scenario, $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ firstly transit into $D_s\eta$ and $D_s^*\eta$, and then η transits into π^0 by the mixing. In the intermediate process, η obviously is off-shell. The mixing depends on the mass difference of η and π, and the effects due to the mixing between η' and π can be ignored.

Another sizable mode is the radiative decay. Even though, by general considerations, the electromagnetic reaction should be much less important than the strong decay, it does not suffer the suppression of isospin violation; therefore one may expect that it has a size comparable to the strong processes described above. The relevant decay modes are $D_{sJ}^*(2317) \to D_s^* + \gamma$, $D_{sJ}(2460) \to D_s + \gamma$ and $D_{sJ}(2460) \to D_s^* + \gamma$ and $D_{sJ}(2460) \to D_{sJ}^*(2317) + \gamma$.

Currently the Babar and the Belle collaborations have completed precise measurements on the ratio of $\Gamma(D_{sJ}(2460) \to D_s\gamma)$ to $\Gamma(D_{sJ}(2460) \to D_s^*\pi^0)$ [3, 32, 33]. Now the Babar and Belle collaborations begin to further measure other decays of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$. We are expecting new results of Babar and Belle which can be applied to decisively determine the structures of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$.

This paper is organized as follows: after the introduction, in Sect. 2, we formulate the strong and radiative decays of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$. In Sect. 3, we present our numerical results along with all the input parameters. Finally, Sect. 4 is devoted to a discussion and to conclusions. Some detailed expressions are collected in the appendix.

2 Formulation

First, for the reader's convenience, we present a brief introduction of the constituent quark meson (CQM) model [26]. The model is relativistic and based on an effective Lagrangian which combines the heavy-quark effective theory (HQET) and the chiral symmetry for light quarks,

$$\begin{aligned} L_{\mathrm{CQM}} = & \bar{\chi}[\gamma(\mathrm{i}\partial + \mathcal{V})]\chi + \bar{\chi}\gamma\mathcal{A}\gamma_5\chi - m_q\bar{\chi}\chi \\ & + \frac{f_\pi^2}{8}\mathrm{Tr}\left[\partial^\mu\Sigma\partial_\mu\Sigma^+\right] + \bar{h}_v(\mathrm{i}v\partial)h_v \\ & - \left[\bar{\chi}\left(\bar{H} + \bar{S} + \mathrm{i}\bar{T}^\mu\frac{\partial_\mu}{\Lambda}\right)h_v + \mathrm{h.c.}\right] \\ & + \frac{1}{2G_3}\mathrm{Tr}[(\bar{H}+\bar{S})(H-S)] + \frac{1}{2G_4}\mathrm{Tr}\left[\bar{T}^\mu T_\mu\right] , \end{aligned} \tag{1}$$

where the fifth term is the kinetic term of heavy quarks with $\not{v}h_v = h_v$; H is the super-field corresponding to the doublet $(0^-, 1^-)$ of negative parity and has the explicit matrix representation

$$H = \frac{1+\not{v}}{2}\left(P_\mu^*\gamma^\mu - P\gamma_5\right) ,$$

where P and $P^{*\mu}$ are the annihilation operators of pseudoscalar and vector mesons which are normalized by

$$\langle 0|P|M(0^-)\rangle = \sqrt{M_H}\,, \text{ and } \langle 0|P^{*\mu}|M(1^-)\rangle = \sqrt{M_H}\epsilon^\mu .$$

S is for the super-fields related to $(0^+, 1^+)$,

$$S = \frac{1+\not{v}}{2}\left[P_{1\mu}^{*'}\gamma^\mu\gamma_5 - P_0\right] .$$

$\chi = \xi q$ ($q = u, d, s$) is the light-quark field and $\xi = \mathrm{e}^{\frac{\mathrm{i}\mathcal{M}}{f_\pi}}$, and M is the octet pseudoscalar matrix. We also have

$$\mathcal{V}^\mu = \frac{1}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right)$$

and

$$\mathcal{A}^\mu = \frac{-\mathrm{i}}{2}\left(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger\right) .$$

Because the spin–parity of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ are 0^+ and 1^+, thus $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ can be embedded into the S-type doublet $(0^+, 1^+)$ [29], whereas D_s and D_s^* belong to the H-type doublet $(0^-, 1^-)$. Then we can calculate the strong and radiative decay rates of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ in the CQM model.

2.1 The transition amplitude of $D_{sJ}^*(2317) \to D_s + \pi^0$ and $D_{sJ}(2460) \to D_s^* + \pi^0$ strong decays in the CQM model

As discussed above indirect $D_{sJ}^*(2317) \to D_s + \pi^0$ and $D_{sJ}(2460) \to D_s^* + \pi^0$ reactions occur via two steps. In Fig. 1, we show the Feynman diagrams which depict the strong processes $D_{sJ}^*(2317) \to D_s + \eta \to D_s + \pi^0$ and $D_{sJ}(2460) \to D_s^* + \eta \to D_s^* + \pi^0$. According to chiral symmetry, η only couples with a light quark; thus it can only be emitted from the light-quark leg in Fig. 1.

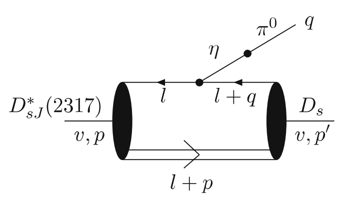

Fig. 1. The Feynman diagrams which depict the decays of $D_{sJ}^*(2317) \to D_s + \pi^0$ or $D_{sJ}(2460) \to D_s^* + \pi^0$. The *double line* denotes the heavy quark (c-quark) propagator

The matrix elements of $D_{sJ}^*(2317) \to D_s + \pi^0$ and $D_{sJ}(2460) \to D_s^* + \pi^0$ are written as

$$\mathcal{M}\left[D_{sJ}^*(2317) \to D_s + \pi^0\right] = \langle\pi^0|\mathcal{L}_{\text{mixing}}|\eta\rangle \frac{\mathrm{i}}{m_\eta^2 - m_\pi^2} \times \langle\eta D_s|\mathcal{L}_{\text{CQM}}|D_{sJ}^*(2317)\rangle\,, \tag{2}$$

$$\mathcal{M}\left[D_{sJ}(2460)) \to D_s^* + \pi^0\right] = \langle\pi^0|\mathcal{L}_{\text{mixing}}|\eta\rangle \frac{\mathrm{i}}{m_\eta^2 - m_\pi^2} \times \langle\eta D_s^*|\mathcal{L}_{\text{CQM}}|D_{sJ}(2460)\rangle\,. \tag{3}$$

The mixing mechanism is described by the Lagrangian[1]

$$\mathcal{L}_{\text{mixing}} = \frac{m_\pi^2}{2\sqrt{3}} \frac{\tilde{m}_u - \tilde{m}_d}{\tilde{m}_u + \tilde{m}_d} \pi^0 \eta\,,$$

which originates from the mass term of the low energy Lagrangian for the pseudoscalar octet [31]

$$\mathcal{L}_{\text{mass}} = \frac{m_\pi^2 f_\pi^2}{4(\tilde{m}_u + \tilde{m}_d)} \text{Tr}\left[\xi m_q \xi + \xi^\dagger m_q \xi^\dagger\right]\,, \tag{4}$$

where m_q is the light-quark mass matrix. The matrix elements of $\langle\eta D_s|\mathcal{H}_{\text{CQM}}|D_{sJ}^*(2317)\rangle$ and $\langle\eta D_s^*|\mathcal{H}_{\text{CQM}}| D_{sJ}(2460)\rangle$ will be calculated in the CQM model.

$\tilde{m}_i (i = u, d, s)$ are the current quark masses. It is noted that in the CQM model calculations the quark masses (m_q, m_c) which we denote as m_q are constituent quark masses [26], whereas, for the mixing, the concerned masses which we denote as $\tilde{m}_q$ are the current quark masses [31].

According to the CQM model [26], couplings of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ with light and heavy quarks are expressed as

$$\frac{1+\not{v}}{2}\sqrt{Z_S M_1}$$

and

$$\frac{1+\not{v}}{2}\sqrt{Z_S M_2}\not{\epsilon}_1\gamma_5\,,$$

and the couplings of D_s and D_s^* to light and heavy quarks are

$$\frac{1+\not{v}}{2}\sqrt{Z_S M_{D_s}}\gamma_5$$

and

$$\frac{1+\not{v}}{2}\sqrt{Z_S M_{D_s^*}}\not{\epsilon}_2\,,$$

where ϵ_1 and ϵ_2 denote the polarization vectors of $D_{sJ}(2460)$ and D_s^* respectively. M_1 and M_2 are respectively the masses of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$. The concrete expressions of Z_H and Z_S are given in [26] as

$$Z_H^{-1} = (\Delta_H + m_s)\frac{\partial I_3(\Delta_H)}{\partial \Delta_H} + I_3(\Delta_H)\,, \tag{5}$$

$$Z_S^{-1} = (\Delta_S + m_s)\frac{\partial I_3(\Delta_S)}{\partial \Delta_S} + I_3(\Delta_S)\,, \tag{6}$$

$$I_3(a) = \frac{\mathrm{i}N_c}{16\pi^4}\int_{1/\Lambda^2}^{1/\mu^2} \frac{\mathrm{d}y}{y^{3/2}} \exp\left[-y\left(m_s^2 - a^2\right)\right] \times (1 + \text{erf}(a\sqrt{y}))\,, \tag{7}$$

where erf is the error function.

Now, we can write out the transition matrix elements as follows:

$$\begin{aligned}&\langle\eta D_s|\mathcal{H}_{\text{CQM}}|D_{sJ}^*(2317)\rangle \\ &= (-1)\mathrm{i}^6\sqrt{Z_S M_1 Z_H M_{D_s}}\sqrt{\frac{2}{3}}\frac{N_c}{2f_\pi} \\ &\times \int^{reg} \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\text{Tr}\left[(\not{l} + m_s)q^\mu\gamma_\mu\gamma_5(\not{l} + \not{q} + m_s)\gamma_5(1+\not{v})\right]}{(l^2 - m_s^2)\left[(l+q)^2 - m_s^2\right](vl + \Delta_S)}\end{aligned} \tag{8}$$

and

$$\begin{aligned}&\langle\eta D_s^*|\mathcal{H}_{\text{CQM}}|D_{sJ}(2460)\rangle \\ &= (-1)\mathrm{i}^6\sqrt{Z_S M_2 Z_H M_{D_s^*}}\sqrt{\frac{2}{3}}\frac{N_c}{2f_\pi}\int^{reg}\frac{\mathrm{d}^4 l}{(2\pi)^4} \\ &\times \frac{\text{Tr}\left[(\not{l} + m_s)q^\mu\gamma_\mu\gamma_5(\not{l} + \not{q} + m_s)\not{\epsilon}_2(1+\not{v})\gamma_5\not{\epsilon}_1\right]}{(l^2 - m_s^2)\left[(l+q)^2 - m_s^2\right](vl + \Delta_S)}\,,\end{aligned} \tag{9}$$

where $N_c = 3$.

Omitting technical details in the text for saving space, we finally obtain

$$\langle\eta D_s|\mathcal{H}_{\text{CQM}}|D_{sJ}^*(2317)\rangle = \sqrt{Z_S M_1 Z_H M_{D_s}}\left(-\sqrt{\frac{2}{3}}\right)\frac{\mathcal{A}}{2f_\pi}\,, \tag{10}$$

$$\langle\eta D_s^*|\mathcal{H}_{\text{CQM}}|D_{sJ}(2460)\rangle = \sqrt{Z_S M_2 Z_H M_{D_s^*}}\left(-\sqrt{\frac{2}{3}}\right) \times \frac{(\epsilon_1\epsilon_2)\mathcal{A}}{2f_\pi}\,, \tag{11}$$

with

$$\begin{aligned}\mathcal{A} &= 4\left(m_s^2 m_\eta\omega - m_\eta^2 m_s\right)\mathcal{O} - m_\eta^2(\mathcal{O}_1 + \omega\mathcal{O}_2) \\ &+ m_\eta\omega(2\mathcal{O}_3 - \mathcal{O}_4 - \mathcal{O}_5 - 2\mathcal{O}_6)\,,\end{aligned} \tag{12}$$

where $\omega = vv' = \frac{\Delta_S - \Delta_H}{2m_\eta}$ and the concrete expressions of $\mathcal{O}$ and $\mathcal{O}_i$ are listed in the appendix.

[1] Here we ignore the mixing of η and η', because the mixing angle $\theta \sim -11^\circ$ is small and does not much affect our results. Therefore we simply assume that η is η_8 and the contribution of η', as discussed in the text, is neglected in our calculations.

The decay widths of $D_{sJ}^*(2317) \to D_s + \pi^0$ and $D_{sJ}(2460) \to D_s^* + \pi^0$ read

$$\begin{aligned} &\Gamma\left[D_{sJ}^*(2317) \to D_s + \pi^0\right] \\ &= \frac{Z_S Z_H M_{D_s} |\mathbf{p}'|}{1024\pi M_1 f_\pi^2} \left(\frac{\tilde{m}_u - \tilde{m}_d}{\tilde{m}_s - (\tilde{m}_u + \tilde{m}_d)/2}\right)^2 |\mathcal{A}|^2 , \end{aligned} \tag{13}$$

$$\begin{aligned} &\Gamma\left[D_{sJ}(2460) \to D_s^* + \pi^0\right] \\ &= \frac{Z_S Z_H M_{D_s} |\mathbf{p}'|}{3072\pi M_2 f_\pi^2} \left(\frac{\tilde{m}_u - \tilde{m}_d}{\tilde{m}_s - (\tilde{m}_u + \tilde{m}_d)/2}\right)^2 \\ &\times \left[2 + \frac{\left(M_2^2 + M_{D_s^*}^2\right)^2}{M_2^2 M_{D_s^*}^2}\right] |\mathcal{A}|^2 , \end{aligned} \tag{14}$$

where the relations $m_\pi^2 = 2\tilde{m}B_0$ and $m_\eta^2 = \frac{2}{3}(\tilde{m} + 2\tilde{m}_s)B_0$ with $\tilde{m} = (\tilde{m}_u + \tilde{m}_d)/2$ are employed to derive the isospin suppression factor $(\tilde{m}_u - \tilde{m}_d)/[\tilde{m}_s - (\tilde{m}_u + \tilde{m}_d)/2]$ [36]. These relations are valid at the leading order of the chiral theory, but the principle does not really apply for estimating the mass of η' (or η_0) due to an extra contribution from the axial anomaly [34].

2.2 The transition amplitude of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ radiative decays in the CQM model

The Feynman diagrams of $D_{sJ}^*(2317) \to D_s^* + \gamma$, $D_{sJ}(2460) \to D_s + \gamma$, $D_{sJ}(2460) \to D_s^* + \gamma$ and $D_{sJ}(2460) \to D_{sJ}^*(2317) + \gamma$ are presented in Fig. 2.

(a) $D_{sJ}^*(2317) \to D_s^* + \gamma$

In contrast with the case in Fig. 1, the heavy quark also couples to γ. Thus the transition matrix element of $D_{sJ}^*(2317) \to D_s^* + \gamma$ is written as *

$$\begin{aligned} &\mathcal{M}\left[D_{sJ}^*(2317) \to D_s^* + \gamma\right] \\ &= (-1)\mathrm{i}^6 \sqrt{Z_S M_1 Z_H M_{D_s^*}} N_c \left\{ \left(\frac{-e}{3}\right) \right. \\ &\times \int^{\mathrm{reg}} \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\mathrm{Tr}\left[(\not{l} + m_s)\not{\epsilon}_\gamma(\not{l} + \not{q} + m_s)\not{\epsilon}_2 \frac{(1+\not{v})}{2}\right]}{(l^2 - m_s^2)\left[(l+q)^2 - m_s^2\right](vl + \Delta_S)} \\ &+ \left. \frac{2e}{3} \int^{\mathrm{reg}} \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\mathrm{Tr}\left[(\not{l} + m_s)\not{\epsilon}_2 \frac{(1+\not{v})}{2} \not{\epsilon}_\gamma \frac{(1+\not{v})}{2}\right]}{(l^2 - m_s^2)(vl + \Delta_H)(vl + \Delta_S)} \right\} \\ &= \sqrt{Z_S M_1 Z_H M_{D_s^*}} \left(\frac{2e}{3}\right) \left\{ (\epsilon_\gamma \epsilon_2) \right. \\ &\times \left[m_s^2 \mathcal{R} - \mathcal{R}_3 - 2\mathcal{R}_6 - \frac{(p'q)(m_s \mathcal{R} + \mathcal{R}_1 + 2\mathcal{R}_5)}{M_{D_s^*}} \right] \\ &+ \left. (\epsilon_\gamma p')(\epsilon_2 q) \left[\frac{m_s \mathcal{R} + \mathcal{R}_1 + 2\mathcal{R}_5}{M_{D_s^*}} \right] \right\} . \end{aligned} \tag{15}$$

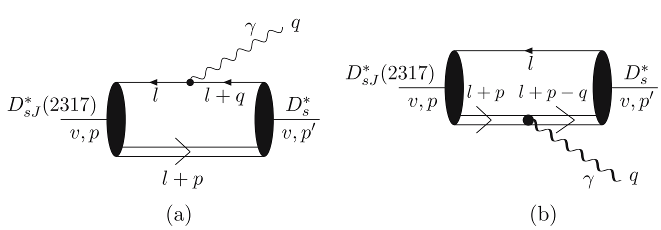

Fig. 2. The Feynman diagrams which depict the radiative decays of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$

(b) $D_{sJ}(2460) \to D_s + \gamma$

The transition matrix element of $D_{sJ}(2460) \to D_s + \gamma$ is

$$\begin{aligned} &\mathcal{M}[D_{sJ}(2460) \to D_s + \gamma] \\ &= (-1)\mathrm{i}^6 \sqrt{Z_S M_2 Z_H M_{D_s}} N_c \left\{ \left(\frac{-e}{3}\right) \int^{\mathrm{reg}} \frac{\mathrm{d}^4 l}{(2\pi)^4} \right. \\ &\times \frac{\mathrm{Tr}\left[(\not{l} + m_s)\not{\epsilon}_\gamma(\not{l} + \not{q} + m_s + \mathrm{i}\epsilon)\gamma_5 \frac{(1+\not{v})}{2} \gamma_5 \not{\epsilon}_1\right]}{(l^2 - m_s^2)\left[(l+q)^2 - m_s^2\right](vl + \Delta_S)} \\ &+ \left. \frac{2e}{3} \int^{\mathrm{reg}} \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\mathrm{Tr}\left[(\not{l} + m_s)\gamma_5 \frac{(1+\not{v})}{2} \not{\epsilon}_\gamma \frac{(1+\not{v})}{2} \gamma_5 \not{\epsilon}_1\right]}{(l^2 - m_s^2)(vl + \Delta_H)(vl + \Delta_S)} \right\} \\ &= \sqrt{Z_S M_1 Z_H M_{D_s}} \left(\frac{2e}{3}\right) \left\{ (\epsilon_\gamma \epsilon_1) \right. \\ &\times \left[m_s^2 \mathcal{R} - \mathcal{R}_3 - 2\mathcal{R}_6 - \frac{(p'q)(m_s \mathcal{R} + \mathcal{R}_1 + 2\mathcal{R}_5)}{M_{D_s}} \right] \\ &+ \left. (\epsilon_\gamma p')(\epsilon_1 q) \left[\frac{m_s \mathcal{R} + \mathcal{R}_1 + 2\mathcal{R}_5}{M_{D_s}} \right] \right\} . \end{aligned} \tag{16}$$

2.2.1 (c) $D_{sJ}(2460) \to D_s^* + \gamma$

The transition matrix element of $D_{sJ}(2460) \to D_s^* + \gamma$ is

$$\begin{aligned} &\mathcal{M}[D_{sJ}(2460) \to D_s^* + \gamma] \\ &= (-1)\mathrm{i}^6 \sqrt{Z_S M_2 Z_H M_{D_s^*}} N_c \left\{ \left(\frac{-e}{3}\right) \right. \\ &\times \int^{\mathrm{reg}} \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\mathrm{Tr}\left[(\not{l} + m_s)\not{\epsilon}_\gamma(\not{l} + \not{q} + m_s)\not{\epsilon}_2 \frac{(1+\not{v})}{2} \gamma_5 \not{\epsilon}_1\right]}{(l^2 - m_s^2)\left[(l+q)^2 - m_s^2\right](vl + \Delta_S)} \\ &+ \left. \frac{2e}{3} \int^{\mathrm{reg}} \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\mathrm{Tr}[(\not{l} + m_s)\not{\epsilon}_2 \frac{(1+\not{v})}{2} \not{\epsilon}_\gamma \frac{(1+\not{v})}{2} \gamma_5 \not{\epsilon}_1]}{(l^2 - m_s^2)(vl + \Delta_H)(vl + \Delta_S)} \right\} \\ &= \sqrt{Z_S M_2 Z_H M_{D_s^*}} \left(\frac{2e}{3}\right) \varepsilon_{\alpha\beta\rho\sigma} q^\alpha \epsilon_\gamma^\beta \epsilon_1^\rho \epsilon_2^\sigma \\ &\times \left[\mathcal{R} m_s - \mathcal{R}_1 + \frac{2\mathcal{R}_2 (p'q)}{M_{D_s^*}} + \frac{\mathcal{R} m_s^2 - \mathcal{R}_3 - 2\mathcal{R}_6}{M_{D_s^*}} \right. \\ &+ \left. \frac{(2\mathcal{R}_5 + 2\mathcal{R}_1)(p'q)}{M_{D_s^*}^2} \right] . \end{aligned} \tag{17}$$

(d) $D_{sJ}(2460) \to D_{sJ}^*(2317) + \gamma$

The transition matrix element of $D_{sJ}(2460) \to D_{sJ}^*(2317) + \gamma$ is

$$
\begin{aligned}
&\mathcal{M}[D_{sJ}(2460) \to D_{sJ}^*(2317) + \gamma] \\
&= (-1)\mathrm{i}^6 \sqrt{Z_S M_1 Z_S M_2} N_c \Bigg\{ \left(\frac{-e}{3}\right) \\
&\quad \times \int^{\mathrm{reg}} \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\mathrm{Tr}\left[(\not{l} + m_s)\not{\epsilon}_\gamma (\not{l} + \not{q} + m_s)\frac{(1+\not{v})}{2}\gamma_5 \not{\epsilon}_1\right]}{(l^2 - m_s^2)\left[(l+q)^2 - m_s^2\right](vl + \Delta_S)} \\
&\quad + \frac{2e}{6} \int^{\mathrm{reg}} \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\mathrm{Tr}\left[(\not{l} + m_s)\frac{(1+\not{v})}{2}\not{\epsilon}_\gamma \frac{(1+\not{v})}{2}\gamma_5 \not{\epsilon}_1\right]}{(l^2 - m_s^2)(vl + \Delta_S)(vl + \Delta_S)} \Bigg\} \\
&= \sqrt{Z_S M_1 Z_S M_2} \frac{2e}{3} \frac{m_s \mathcal{R}}{M_1} \varepsilon_{\alpha\beta\rho\sigma} q^\alpha p'^\beta \epsilon_\gamma^\rho \epsilon_1^\sigma \,. \qquad (18)
\end{aligned}
$$

The concrete expressions of $\mathcal{R}$ and $\mathcal{R}_i$ are listed in the appendix.

3 Numerical results

With the formulation we derived in last section, we can numerically evaluate the corresponding decay rates. Besides, we need several input parameters for the numerical computations. They include: $f_\pi = 132\,\mathrm{MeV}$, $m_s = 0.5\,\mathrm{GeV}$, $\Lambda = 1.25\,\mathrm{GeV}$, the infrared cutoff $\mu = 0.51\,\mathrm{GeV}$ and $\Delta_S - \Delta_H = 335 \pm 35\,\mathrm{MeV}$ [29]. Also, we have $m_\eta = 547.45\,\mathrm{MeV}$, $M_1 = 2317\,\mathrm{MeV}$, $M_2 = 2460\,\mathrm{MeV}$, $M_{D_s} = 1968\,\mathrm{MeV}$ and $M_{D_s^*} = 2112\,\mathrm{MeV}$ [35]. The suppression parameter was estimated in [36] as

$$
\frac{\tilde{m}_u - \tilde{m}_d}{\tilde{m}_s - (\tilde{m}_u + \tilde{m}_d)/2} \sim \frac{1}{43.7} \,.
$$

We present the decay widths of $D_{sJ}^*(2317) \to D_s + \pi^0$ and $D_{sJ}(2460) \to D_s^* + \pi^0$ in Table 1.

From (5)–(7), one can notice that $Z_{H,S}$ not only depend on $\Delta_{H,S}$ but also on the choice of the two energy scales Λ and μ. The authors of [29], discussed how to determine the energy scale. In this work we take the values given in [29], i.e. $\mu = 0.51\,\mathrm{GeV}$ and $\Lambda = 1.25\,\mathrm{GeV}$; then we let μ vary to 0.4 GeV, (20%), and we find that the change of Z_H and Z_S is less than 10%. It indicates that Z_H and Z_S are not very sensitive to the change of the energy scale μ. The parameter Λ is the chiral symmetry energy scale and usually is taken around 1 GeV.

For a comparison, we also list the values of the decay widths of $D_{sJ}^*(2317) \to D_s + \pi^0$ and $D_{sJ}(2460) \to D_s^* + \pi^0$, which are calculated by other groups, in Table 2.

Our values depend on the model parameters, but qualitatively, the order of magnitude is unchanged when the parameters vary within a reasonable region. All the values in Table 2 are somehow consistent with each other as regards order of magnitude, even though there is an obvious difference in numbers.

With the same parameters, we obtain the radiative decay rates of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$. The results are listed in Table 3.

For convenience, we define the relevant ratios as

$$
\begin{aligned}
R_1 &= \Gamma(D_{sJ}^*(2317) \to D_s + \gamma) : \Gamma\left(D_{sJ}^*(2317) \to D_s + \pi^0\right) \,, \\
R_2 &= \Gamma(D_{sJ}(2460) \to D_s + \gamma) : \Gamma\left(D_{sJ}(2460) \to D_s^* + \pi^0\right) \,, \\
R_3 &= \Gamma(D_{sJ}(2460) \to D_s^* + \gamma) : \Gamma\left(D_{sJ}(2460) \to D_s^* + \pi^0\right) \,, \\
R_4 &= \Gamma\left(D_{sJ}(2460) \to D_{sJ}^*(2317) + \gamma\right) : \Gamma \\
&\quad \times \left(D_{sJ}(2460) \to D_s^* + \pi^0\right) \,.
\end{aligned}
$$

Thus we calculate the R_i's and tabulate the results below in Table 4.

4 Conclusion and discussion

In this work, based on the assumption that $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ are chiral parters of D_s and D_s^*, we calculate the rates of $D_{sJ}^*(2317) \to D_s\pi^0$ and $D_{sJ}(2460) \to D_s^*\pi^0$ in the constituent quark meson (CQM) model and take into account the η–π^0 mixing mechanism [31]. We also estimate the rates of $D_{sJ}^*(2317) \to D_s^* + \gamma$, $D_{sJ}(2460) \to D_s + \gamma$, $D_{sJ}(2460) \to D_s^* + \gamma$ and $D_{sJ}(2460) \to D_{sJ}^*(2317) + \gamma$ in the same model.

Comparing our results for the strong decay rates with those obtained by other groups, we find that our results are reasonably consistent with the values listed in Table 2.

For the radiative decay, our results generally coincide with that obtained by other groups and especially these results of the QCD sum rules.

The ratio R_2 has been measured with relatively high precision [3, 32, 33]; however for R_1, R_3 and R_4, there only are upper limits given by the Babar, Belle, and CLEO collaborations [2, 32, 33]. It seems that our results on the ratios well coincide with the experimental values. This consistency somehow implies that the assumption of $D_{sJ}^*(2317)$, $D_{sJ}(2460$ being p-wave chiral partners of D_s, D_s^* does not contradict the data with the present experimental accuracy.

The experimental upper bounds on the total widths are $\Gamma(D_{sJ}^*(2317)) < 4.6\,\mathrm{MeV}$ and $\Gamma(D_{sJ}(2460)) < 5.5\,\mathrm{MeV}$ [35]. Obviously, the overwhelming decay modes

Table 1. The values of Δ_S and Δ_H are taken from [29]. According to (5) and (6), one gets the values of Z_S and Z_H. Γ_1 and Γ_2 denote respectively the decay widths of $D_{sJ}^*(2317) \to D_s + \pi^0$ and $D_{sJ}(2460) \to D_s^* + \pi^0$

Δ_H (GeV)	Δ_S (GeV)	Z_H $(\mathrm{GeV})^{-1}$	Z_S $(\mathrm{GeV})^{-1}$	Γ_1 (keV)	Γ_2 (keV)
0.5	0.86	3.99	2.02	3.68	1.86
0.6	0.91	2.69	1.47	5.36	2.72
0.7	0.97	1.74	0.98	8.71	4.42

Table 2. In this table, we list our calculation of $D^*_{sJ}(2317) \to D_s + \pi^0$ and $D_{sJ}(2460) \to D^*_s + \pi^0$ and that obtained by other groups, where Γ_1 and Γ_2 respectively denote the decay widths of $D^*_{sJ}(2317) \to D_s + \pi^0$ and $D_{sJ}(2460) \to D^*_s + \pi^0$

	CQM model	[14]	[15]	[16]	[17]	[18]	[19]
Γ_1 (keV)	$3.68 \sim 8.71$	6 ± 2	$34 \sim 44$	7 ± 1	21.5	~ 10	16
Γ_2 (keV)	$1.86 \sim 4.42$	–	$35 \sim 51$	7 ± 1	21.5	~ 10	32

of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ are their strong and radiative decays, therefore we can roughly take the sums of these widths as the total widths of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$. However, our numerical results as well as those given by other groups are in order of tens of keV, much smaller than the upper bounds set by recent experiments. The reason is obvious that the aforementioned reactions violate isospin conservation; there is a large suppression factor of about $(1/43.7)^2$, which reduces the widths by 3 orders. The calculations which are based on the assumption that the newly discovered $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ are p-wave excited states of D_s and D^*_s predict their widths to be at order of a few to tens of keV. By contrast, if they are four-quark states, or molecular states, there may be more decay channels available, i.e. some modes are not constrained by the OZI rule, thus much larger total widths might be expected in that scenario. The authors of [9, 21], for example, suggested that $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ are in a non-$c\bar{s}$ structure (four-quark states etc.), and obtained much larger rates, even though still obviously smaller than the experimental upper bounds. So far, it is hard to conclude if they are p-wave chiral partners of D_s and D^*_s or four-quark states yet.

We hope that the further more precise measurements of Babar, Belle and CLEO may offer more information by which we may determine the structure of the newly discovered mesons.

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Appendix

We have

$$\mathcal{O} = \frac{I_5(\Delta_S, m_\eta/2, \omega) - I_5(\Delta_H, -m_\eta/2, \omega)}{2m_\eta}, \tag{A.1}$$

$$\mathcal{O}_1 = \frac{I_3(-m_\eta/2) - I_3(m_\eta/2) + \omega[I_3(\Delta_S) - I_3(\Delta_H)]}{2m_\eta(1-\omega^2)} - \frac{\mathcal{O}[\Delta_S - \omega m_\eta/2]}{1-\omega^2}, \tag{A.2}$$

$$\mathcal{O}_2 = \frac{-I_3(\Delta_S) + I_3(\Delta_H) - \omega[I_3(-m_\eta/2) - I_3(m_\eta/2)]}{2m_\eta(1-\omega^2)} - \frac{\mathcal{O}[m_\eta/2 - \Delta_S\omega]}{1-\omega^2}, \tag{A.3}$$

$$\mathcal{O}_3 = \frac{\mathcal{B}_1}{2} + \frac{2\omega\mathcal{B}_4 - \mathcal{B}_2 - \mathcal{B}_3}{2(1-\omega^2)^2}, \tag{A.4}$$

$$\mathcal{O}_4 = \frac{-\mathcal{B}_1}{2(1-\omega^2)} + \frac{3\mathcal{B}_2 - 6\omega\mathcal{B}_4 + \mathcal{B}_3(2\omega^2+1)}{2(1-\omega^2)^2}, \tag{A.5}$$

$$\mathcal{O}_5 = \frac{-\mathcal{B}_1}{2(1-\omega^2)} + \frac{3\mathcal{B}_3 - 6\omega\mathcal{B}_4 + \mathcal{B}_2(2\omega^2+1)}{2(1-\omega^2)^2}, \tag{A.6}$$

$$\mathcal{O}_6 = \frac{\mathcal{B}_1\omega}{2(1-\omega^2)} + \frac{2\mathcal{B}_4(2\omega^2+1) - 3\omega(\mathcal{B}_2 + \mathcal{B}_3)}{2(1-\omega^2)^2}, \tag{A.7}$$

$$\mathcal{B}_1 = m_s^2\mathcal{O} - I_3(\Delta_H), \tag{A.8}$$

$$\mathcal{B}_2 = \Delta_S^2\mathcal{O} - \frac{I_3(m_\eta/2) - I_3(-m_\eta/2)}{4m_\eta} \times (\omega m_\eta + 2\Delta_S), \tag{A.9}$$

$$\mathcal{B}_3 = \frac{m_\eta^2\mathcal{O}}{4} + \frac{I_3(\Delta_S) - 3I_3(\Delta_H)}{4}$$

Table 3. The rates of $D^*_{sJ}(2317) \to D_s + \gamma$, $D_{sJ}(2460) \to D_s + \gamma$, $D_{sJ}(2460) \to D^*_s + \gamma$ and $D_{sJ}(2460) \to D^*_{sJ}(2317) + \gamma$. We also list the results obtained by other groups

	CQM model	[20]	[16]	[17]	[18]
$\Gamma\,(D^*_{sJ}(2317) \to D_s + \gamma)$ (keV)	~ 1.1	$4 \sim 6$	0.85	1.74	1.9
$\Gamma\,(D_{sJ}(2460) \to D_s + \gamma)$ (keV)	$0.6 \sim 2.9$	$19 \sim 29$	3.3	5.08	6.2
$\Gamma\,(D_{sJ}(2460) \to D^*_s + \gamma)$ (keV)	$0.54 \sim 1.4$	$0.6 \sim 1.1$	1.5	4.66	5.5
$\Gamma\,(D_{sJ}(2460) \to D^*_{sJ}(2317) + \gamma)$ (keV)	$0.13 \sim 0.22$	$0.5 \sim 0.8$	–	2.74	0.012

Table 4. The ratios of radiative decay widths to strong decay widths for $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$. The first three columns are the experimental data and the fourth column is our result calculated in the CQM model

	Belle	Babar	CLEO [2]	CQM model
R_1	< 0.18 [32]	–	< 0.059	$0.12 \sim 0.30$
R_2	$0.55 \pm 0.13 \pm 0.08$ [32]	$0.375 \pm 0.054 \pm 0.057$ [33]	< 0.49	$0.32 \sim 0.66$
R_3	< 0.31 [32]	–	< 0.16	$0.29 \sim 0.32$
R_4	–	< 0.23 [33]	< 0.58	$0.05 \sim 0.07$

$$+\frac{\omega}{4}[\Delta_S I_3(\Delta_S)-\Delta_H I_3(\Delta_H)]\,, \tag{A.10}$$

$$\mathcal{B}_4=\frac{m_\eta\Delta_S\mathcal{O}}{2}+\frac{\Delta_S[I_3(\Delta_S)-I_3(\Delta_H)}{2m_\eta}+\frac{I_3(m_\eta/2)-I_3(-m_\eta/2)}{4}\,, \tag{A.11}$$

$$I_5(a_2,a_3,a_4)=\int_0^1 \mathrm{d}x\frac{1}{1+2x^2(1-a_4)+2x(a_4-1)}\times\left\{\frac{6}{16\pi^{3/2}}\int_{1/\Lambda^2}^{1/\mu^2}\frac{\mathrm{d}y}{\sqrt{y}}\xi\exp\left[-y\left(m_s^2-\xi^2\right)\right]\times[1+\mathrm{erf}(\xi\sqrt{y})]+\frac{6}{16\pi^2}\times\int_{1/\Lambda^2}^{1/\mu^2}\frac{\mathrm{d}y}{y}\exp\left[-y\left(m_s^2-2\xi^2\right)\right]\right\}, \tag{A.12}$$

$$\xi=\frac{a_2(1-x)+a_3x}{\sqrt{1+2(a_4-1)x+2(1-a_4)x^2}}\,, \tag{A.13}$$

$$\mathcal{R}=\frac{3}{16\pi^{3/2}}\int_{1/\Lambda^2}^{1/\mu^2}\mathrm{d}s\frac{exp\left[-sm_s^2\right]}{s^{1/2}}\times\int_0^1\mathrm{d}x\exp[s\Delta^2(x)][1+\mathrm{erf}(\Delta(x)\sqrt{s})]\,, \tag{A.14}$$

$$\mathcal{R}_1=\frac{n_2}{|\mathrm{q}|}\,, \tag{A.15}$$

$$\mathcal{R}_2=\frac{n_1-\mathcal{R}_1}{|\mathrm{q}|}\,, \tag{A.16}$$

$$\mathcal{R}_3=\frac{n_4}{|\mathrm{q}|^2}\,, \tag{A.17}$$

$$\mathcal{R}_6=\frac{\mathcal{R}_3+n_6}{2}-\frac{n_5}{|\mathrm{q}|}\,, \tag{A.18}$$

$$\mathcal{R}_5=\frac{n_5}{|\mathrm{q}|^2}-\frac{\mathcal{R}_3+\mathcal{R}_6}{|\mathrm{q}|}\,, \tag{A.19}$$

$$\mathcal{R}_4=\frac{n_3-\mathcal{R}_3-\mathcal{R}_6}{|\mathrm{q}|^2}-\frac{2\mathcal{R}_5}{|\mathrm{q}|}\,, \tag{A.20}$$

$$\Delta(x)=\Delta_S-x|\mathrm{q}|,\quad I_2=\frac{3}{16\pi^2}\Gamma\left(0,\frac{m_s^2}{\Lambda^2},\frac{m_s^2}{\mu^2}\right) \tag{A.21}$$

$$n_1=-I_2-\Delta_S\mathcal{R}\,, \tag{A.22}$$

$$n_2=\frac{1}{2}[-I_3(\Delta_S)+I_3(\Delta_H)]\,, \tag{A.23}$$

$$n_3=\Delta_S^2\mathcal{R}+\left(\frac{|\mathrm{q}|}{2}+\Delta_S\right)I_2\,, \tag{A.24}$$

$$n_4=\frac{|\mathrm{q}|}{2}[\Delta_S I_3(\Delta_S)-\Delta_H I_3(\Delta_H)]\,, \tag{A.25}$$

$$n_5=\frac{\Delta_S}{2}[I_3(\Delta_S)-I_3(\Delta_H)]\,, \tag{A.26}$$

$$n_6=m_s^2\mathcal{R}-I_3(\Delta_H)\,, \tag{A.27}$$

where q is the three-momentum of the emitted photon.

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